

# Quark Pair Production in Expanding Glasma

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Glasma in high energy heavy ion collisions is longitudinal classical color electric and magnetic fields. The color electric field has been shown to produce quark and anti-quark pairs by the Schwinger mechanism and to oscillate with time in non-expanding glasma, that is, plasma oscillation. On the other hand, in the expanding glasma we show that the field decreases with the plasma oscillation. We can explicitly obtain the solutions representing such temporal behaviors in the system with  $\tau$  and  $\eta$  coordinates. We show these results by using massless QED as a simplified model of QCD.

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## I. INTRODUCTION

Initial states of color gauge fields (glasma) produced immediately after high energy heavy-ion collisions have recently received much attention. The gauge fields are longitudinal classical color electric  $E$  and magnetic  $B$  fields. The presence of such classical gauge fields has been discussed on the basis of a fairly reliable effective theory of QCD at high energy, that is, a model of color glass condensate (CGC)[1, 2]. The glasma is generated by small  $x$  gluons in nuclei. It is expected that the decay of the glasma leads to thermalized quark gluon plasma (QGP). The study of the decay of the glasma is very significant because the classical and quantum behaviors of the coherent non-Abelian gauge fields have not been experimentally known.

The glasma is homogeneous in the longitudinal direction but inhomogeneous in the transverse directions. Hence, we may view that it forms electric and magnetic flux tubes extending in the longitudinal direction. Their field strengths are given by the saturation momentum  $Q_s$  in high energy heavy ion collisions such that  $eB \sim Q_s^2$  and  $eE \sim Q_s^2$ ;  $e(>0)$  denotes the coupling constant. In the previous papers[3–5] we have shown a possibility that the famous Nielsen-Olesen instability[6] makes the color magnetic field  $B$  decay. The possibility has been partially confirmed by the comparison between our results and numerical simulations[7, 8]. Such decay is a first step toward the generation of QGP. (The gauge fields produced initially in the model of color glass condensate have very random spatial distribution without no typical length scales[9]. The Nielsen-Olesen unstable modes do not arise under such random configurations of the fields. But soon after the production they would rapidly evolve to form a smooth distribution with the typical length scale  $Q_s^{-1}$  owing to the nonlinear interactions among the fields[10]. Then, the unstable modes can arise under the smooth background gauge fields. )

Furthermore, we have also discussed[11–13] the decay of the color electric field; the decay is caused by Schwinger mechanism[14], that is, the pair production of quarks and anti-quarks. The mechanism has been extensively explored[15] since the discovery of the Klein paradox. A new feature in the glasma is that it is composed of not only electric field but also magnetic field collinear to the electric field. Under such a circumstance there are only several studies of the mechanism involving the back reaction of the produced particles on the electric field[12, 15, 16]. It has been found that the number density of quarks and the electric field oscillate in time, namely the plasma oscillation when the quarks are free after their production. On the other hand, it has recently shown[17] that when the quarks interact with each other in heat bath, the electric field decays without the oscillation, while their number density increases and is saturated. These results have been obtained in a system of non-expanding glasma using QED for simplicity.

In this paper we analyze the behaviors of the electric field and the quark number density in the expanding glasma. We only consider the case of non-interacting quarks after their production. The pair production in the expanding glasma has already been discussed[18], but there is still no analysis in the case that both electric and magnetic fields are present. In order to describe the expanding glasma, we choose a coordinate system such as  $\tau = \sqrt{t^2 - z^2}$  and  $\eta = \log \sqrt{(t+z)/(t-z)}$  instead of  $t$  and  $z$  in Cartesian coordinate. (  $\tau$  is a proper time associated with a particle whose longitudinal coordinate is specified by  $\eta = \text{constant}$ . ) For simplicity, we analyze U(1) gauge theory instead of SU(3) gauge theory. Thus, we use the terminology of electrons and positrons. They are assumed to be massless particles. This is because the corresponding masses of quarks can be neglected in the glasma. The typical scales in the glasma is given by the saturation momentum  $Q_s$ , which is much larger than the mass of the quarks ;

$Q_s \simeq 1\text{GeV} \sim 2\text{GeV}$  for RICH or LHC. We also assume that only relevant states in the pair production are those in the lowest Landau level with their energies given by  $|p_z|$  in the Cartesian coordinate where  $p_z$  is the momentum parallel to  $\vec{B}$ . The assumption holds only in the limit of  $eB \rightarrow \infty$ , although both  $eB$  and  $eE$  are of the order of  $Q_s^2$  in the real glasma. We discuss that the assumption is approximately correct even when  $eB$  and  $eE$  are the same order of magnitudes as each other. We find that both the electron number density and the electric field decrease oscillating with time. The decrease is caused by the expansion of glasma.

In order to show these results, we use the chiral anomaly. The chiral anomaly is a quite powerful tool for the discussion of the Schwinger mechanism. It is effective only when both collinear electric and magnetic fields are present. When the magnetic field is very strong, the produced particles occupy only the states in the lowest Landau level. It implies that the transverse motion of the particles are frozen and only the longitudinal motion is allowed. Thus, the particle production effectively takes place in two dimensional space-time. Then, the form of the chiral anomaly is much simplified. The pair production can be discussed without detail calculations but simply by solving the anomaly equation.

In the next section II we explain how the pair production effectively arises in two dimensional space-time when the magnetic field is very strong. We define a two dimensional field operator using creation and annihilation operators of the states in the lowest Landau level. Electric charge and chiral current of the states are represented with the field operator. The analysis in this section is performed in the Cartesian coordinate. In the section III we show the utility of the chiral anomaly for the pair production by using the field operator. In the section IV we apply our method to analyse the pair production in  $\tau$  and  $\eta$  coordinates. We find that the electric field decreases oscillating with time owing to the back reaction and the expansion of the glasma. We summarize our results in the section V.

## II. THE LOWEST LANDAU LEVEL AND REDUCTION OF SPATIAL DIMENSIONS

First, we explain solutions of Dirac equation under the homogeneous magnetic field in the Cartesian coordinate and the reduction of the four dimensional operators to the two dimensional ones. We use the gauge potential  $\vec{A} = (0, xB, 0)$  of the magnetic field  $\vec{B} = (0, 0, B)$ . Then, the solutions representing massless fermions in the lowest Landau level which we are only concerned with, are given in the following,

$$\Psi = N_0 \exp(-iE_p t + ip_y y + ip_z z) \exp\left(-\frac{eB}{2}\left(x - \frac{p_y}{eB}\right)^2\right) u(p) \quad \text{with} \quad u(p) = \begin{pmatrix} 1 \\ 0 \\ p_z/E_p \\ 0 \end{pmatrix} \quad (1)$$

where  $E_p = \pm|p_z|$  represents the energy and  $N_0$  denotes a normalization constant. On the other hand, the states in higher Landau levels have the energies  $\pm\sqrt{enB + p_z^2}$  where the integer  $n \geq 1$  denotes Landau level. Thus, in the limit of  $eB \rightarrow \infty$ , only relevant states in the pair production are the states with the energies  $|p_z|$  in the lowest Landau level. ( In addition to the states there are other states in the lowest Landau level whose energies are given by  $\sqrt{eB + p_z^2}$ . The states are those with magnetic moments anti-parallel to  $\vec{B}$ , carrying much higher energies than the energies of the above states. These states are also irrelevant in the limit. ) Hereafter we only consider the states in the lowest Landau level, which are the most relevant states in the Schwinger mechanism when  $B \gg E$ .

The transverse motions are described by the factor  $\exp(+ip_y y) \exp(-eB(x - p_y/eB)^2/2)$  of the wave functions in the lowest Landau level. The states are degenerate in the momentum  $p_y$ . The states in higher Landau levels are described by different functions of the transverse coordinates  $x$  and  $y$ . In the sense, the transverse motions are frozen, as long as we are concerned with such states in the lowest Landau level. Only dynamical behaviors of the physical quantities are allowed in the longitudinal direction. Namely, the states in the lowest Landau level are specified with quantum numbers  $p_z$  and  $p_y$ , whose energies are given by  $|p_z|$ . Since the electric field parallel to  $\vec{B}$  accelerates the particles, only the momentum  $p_z$  increases or decreases, but the momentum  $p_y$  does not change. Thus, the momentum  $p_y$  is conserved and the particles do not jump into higher Landau levels as long as their energies  $|p_z| \leq eB$ . ( Although the momentum  $p_z$  increases by the acceleration of the electric field, the electric field decays with the back reaction until  $p_z$  becomes sufficiently large such as  $|p_z| > eB$ . ) Namely, the transverse motions specified by the coordinates  $x$  and  $y$  are frozen. Therefore, we only need to specify the quantum number  $p_z$  in order to describe the states in the lowest Landau level.

Since they are trivial in the transverse directions, we take an average over the transverse space of physical quantities, e.g.  $\bar{\Psi} O \Psi$  with gamma matrices  $O$ . Then, we find that the average over the transverse space of  $\bar{\Psi} O \Psi$  are given by  $\bar{\psi} O \psi$  where the field operator  $\psi$  is defined as,

$$\psi = \int \frac{dp}{4\pi} \left( \exp(-i\omega_p t + ipz) u(p) A_p + \exp(i\omega_p - ipz) u(p) B_p^\dagger \right) \quad (2)$$

with  $\omega_p \equiv |p|$  and  $p \equiv p_z$ , where  $A_p$  and  $B_p$  denote annihilation operators of particles and anti-particles with the momentum  $p$ , respectively, and satisfy the following commutation relations,

$$A_p A_q^\dagger + A_q^\dagger A_p \equiv \{A_p, A_q^\dagger\} = \{B_p, B_q^\dagger\} = 2\pi\delta(p - q), \quad \text{the other commutators vanish.} \quad (3)$$

Then, it follows that

$$\{\psi^\dagger(t, z), \psi(t, z')\} = \delta(z - z'). \quad (4)$$

The field  $\psi$  satisfies effectively 2 dimensional Dirac equation,  $(i\gamma^0 \partial_t + \gamma^3 \partial_z)\psi = 0$  where

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}, \quad \text{and} \quad \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Note that the spinor and the gamma matrices are not two dimensional one. The term of "two dimensional" means that the field depends only on the coordinates  $t$  and  $z$ . When homogeneous electric field  $E = -\partial_t A_3(t)$  is present in addition to the magnetic field, the operators  $A_p$  and  $B_p$  depend on  $t$ , but keep the above commutation relations. They satisfy the equations,  $i\partial_t A_p = eA_3 A_p p/|p|$  and  $i\partial_t B_p^\dagger = eA_3 B_p^\dagger p/|p|$ .

Using the field  $\psi$ , we find that the expectation values of the electric charge  $e\bar{\psi}\gamma^0\psi$ , electric current  $e\bar{\psi}\gamma^3\psi$ , chiral charge  $\bar{\psi}\gamma^0\gamma^5\psi$  and chiral current  $\bar{\psi}\gamma^3\gamma^5\psi$  are given respectively by

$$\begin{aligned} J^0 &\equiv \langle e\bar{\psi}\gamma^0\psi \rangle = e \int \frac{dp}{2\pi} (n_p - \tilde{n}_p), \quad \text{and} \quad J^3 \equiv \langle e\bar{\psi}\gamma^3\psi \rangle = e \int \frac{dp}{2\pi} \frac{p}{\omega_p} (n_p - \tilde{n}_p) \\ J^5 &\equiv \langle \bar{\psi}\gamma^0\gamma^5\psi \rangle = \int \frac{dp}{2\pi} \frac{p}{\omega_p} (n_p - \tilde{n}_p) \quad \text{and} \quad J^{3,5} \equiv \langle \bar{\psi}\gamma^3\gamma^5\psi \rangle = \int \frac{dp}{2\pi} (n_p - \tilde{n}_p), \end{aligned} \quad (5)$$

where operator products are defined as normal ordered product with respect to  $A_p$  and  $B_p$ . We have taken the expectation values of these quantities by assuming appropriate states  $|\rangle$  relevant to the particle production under the electric field, which satisfy

$$\langle A_p^\dagger A_q \rangle = 2\pi\delta(p - q)n_p(t), \quad \langle B_p^\dagger B_q \rangle = 2\pi\delta(p - q)\tilde{n}_p(t) \quad \text{and} \quad \langle A_p^\dagger B_q^\dagger \rangle = \langle B_p A_q \rangle = 0 \quad (6)$$

Obviously  $A_p$  ( $B_p$ ) represents annihilation operators of positrons (electrons) with the electric charge  $e > 0$  ( $-e < 0$ );  $n_p(t)$  represents momentum distribution of the positrons, while  $\tilde{n}_p(t)$  represents that of electrons. Both of them depend on time  $t$ , in general.

We denote the number density of positrons by  $N_p = \int \frac{dp}{2\pi} n_p$  and of electrons by  $N_e = \int \frac{dp}{2\pi} \tilde{n}_p$  respectively. Then, we find that

$$\begin{aligned} J^0 &= eN_p - eN_e, \quad J^{3,5} = N_p - N_e, \\ J^3 &= eJ^5 = e(N_p + N_e) \quad \text{for} \quad \vec{E} \cdot \vec{B} > 0 \quad \text{and} \quad J^3 = eJ^5 = -e(N_p + N_e) \quad \text{for} \quad \vec{E} \cdot \vec{B} < 0. \end{aligned} \quad (7)$$

The electric charge  $J^0$  is given by the number densities of electrons and positrons as expected. On the other hand we need to explain why the form of the chirality  $J^5$  is given as in Eq(7). Because positrons (electrons) are accelerated into the direction parallel (anti-parallel) to  $\vec{E}$ , the momentum of positrons (electrons) is given by  $p > 0$  ( $p < 0$ ) for  $\vec{E} \cdot \vec{B} > 0$  or  $p < 0$  ( $p > 0$ ) for  $\vec{E} \cdot \vec{B} < 0$ ;  $\vec{B} = (0, 0, B)$  with  $B > 0$ . (The particles are spontaneously produced with their momentum equal to 0 because of the Pauli principle.) Then, the helicity of both positrons and electrons is positive for  $\vec{E} \cdot \vec{B} > 0$  or negative for  $\vec{E} \cdot \vec{B} < 0$ , since the spins of the positrons (electrons) are pointed into the direction parallel (anti-parallel) to  $\vec{B}$ . Therefore, the chirality  $J^5$  is given as in Eq(7). This is owing to the fact that the produced particles occupy the lowest Landau level with their magnetic moment parallel to  $\vec{B}$  and their momenta are determined by the electric field.

### III. CHIRAL ANOMALY AND PLASMA OSCILLATION

Next, we explain the utility of the chiral anomaly and how the pair production under homogeneous electric and magnetic fields is described by using the anomaly. Suppose that the magnetic field is sufficiently strong for the charged particles to occupy only the lowest Landau level. Then, we may use the effectively two dimensional formulas as described above. The anomaly equation is given in the following,

$$\partial_t J^5 = \frac{e^2}{2\pi^2} \vec{E}(t) \cdot \vec{B} \quad (8)$$

where we have taken an average over the transverse space  $\vec{x}_T = (x_1, x_2)$ , of the chiral current so that  $\int d\vec{x}_T \partial_{\vec{T}} J^{\vec{T},5} = 0$ . We have also used the relation  $\partial_3 J^{3,5} = 0$ . The term in the right hand side of Eq(8) arises by taking account of quantum effects ( loop diagrams ) of electrons. That is, although the chiral current is conserved classically, the conservation is violated by the quantum effects. It is important to note that the violation term is given by the product of the electric and magnetic fields.

When the electric field  $E$  is switched on at  $t = 0$  in vacuum, the pair production arises and the chiral charge  $J^5$  is produced according to the anomaly equation (8). Suppose that the electric field is parallel to  $B$ . Then, positrons move to the direction parallel to  $E$ , while electrons move to the direction anti-parallel to  $E$ . We have  $n_p \propto \theta(p)$  for positrons and  $\tilde{n}_p \propto \theta(-p)$  for electrons. Thus, it follows from Eq(5) that  $J^5 = N_p + N_e$ . Since  $N_p = N_e$  in the pair production, the anomaly equation becomes

$$\partial_t N = sE \quad (9)$$

with  $N \equiv N_p$  and  $s \equiv e^2 B / 4\pi^2$ . Similarly, when  $E$  is anti-parallel to  $B$ , then  $n_p \propto \theta(-p)$  and  $\tilde{n}_p \propto \theta(p)$ . Thus,  $J^5 = -N_p - N_e$ . Since  $\vec{E} \cdot \vec{B} = -EB$ , the anomaly equation (9) is also obtained. Hence it holds in both cases with  $\vec{E} \cdot \vec{B} = EB$  and  $\vec{E} \cdot \vec{B} = -EB$ .

Obviously, the number density of the electrons and positrons produced by the electric field is governed by the anomaly equation (9). For example, when  $E$  is switched on at  $t = 0$ , the particles are produced so that the number density is given by  $N = sEt$ . This is the result with no back reaction of the charged particles on  $E$ . In order to take into account of the back reaction, we need to solve a Maxwell equation  $\partial_t E = -J^3 = -2eN$  as well as the anomaly equation (9); the Maxwell equation holds in a spatially homogeneous system (  $\text{rot } \vec{B} = 0$  ). From these equations, we derive the equation

$$\partial_t^2 E(t) + 2esE(t) = \partial_t^2 E(t) + \frac{e^3 B}{2\pi^2} E(t) = 0, \quad (10)$$

whose solutions with the initial conditions  $E(t=0) = E_0$  and  $N(t=0) = 0$  are trivially obtained;  $E = E_0 \cos(\sqrt{2es}t)$  and  $N = E_0 \sqrt{s/2e} \sin(\sqrt{2es}t)$ . The electric field shows a plasma oscillation in Fig.1 with the frequency of the order of  $\sqrt{eB}$  (  $\sim Q_s$  in the glasma ). Although we have solved only classical equations, the anomaly equation involves all of quantum effects associated with the pair productions. Thus, the chiral anomaly is very useful tool for the investigation of the Schwinger mechanism when both strong magnetic and electric field are present.

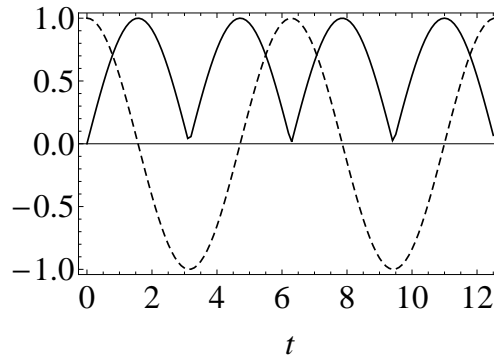


FIG. 1: electric field  $E(t)$  (dashed line) and number density of electrons  $N(t)$  (solid line) with arbitrary scale

We should make a comment on the scales involved in the discussion. The above result ( the plasma oscillation ) holds in the limit of  $eB \gg eE$  where only the states in the lowest Landau level are relevant. On the other hand when  $eB \ll eE$ , the states in higher Landau levels become important since the energy  $|p_F(t)| = |\int_0^t dt' eE|$  of electrons can be much larger than  $\sqrt{eB}$  soon after their production. In the glasma we have  $eB \simeq eE \sim Q_s^2 \gg m_{\text{quark}}^2$  ( the square of quark mass ). We argue that even in such a case the states in higher Landau levels are still not important according to the following reason. Most of the electrons are produced with their energies equal to 0. After their production, they are accelerated and their energies increase. The energies of the electrons produced at  $t = 0$  reach the energy  $|p_F| = \int_0^{Q_s^{-1}} dt' eE \simeq eEQ_s^{-1} \sim \sqrt{eB}$  at the time of  $Q_s^{-1}$ , the typical time scale of the oscillation. Then, they may make a transition to a state in a higher Landau level with the energy  $\sqrt{eB}$ . Such electrons are those which are produced just after the electric field is switched on. Most of electrons still have their energies less than  $\sqrt{eB}$ . So, the number of the electrons whose energies reach the critical energy  $\sqrt{eB}$  is still a fraction. Most of the electrons stay in the lowest Landau level.

It has recently been shown[16] that the vacuum decay by the pair production is mainly caused by massless particles in the lowest Landau level even if  $eE \sim eB$ . Although their analysis does not involve the back reaction of the particles produced, it supports our approximation that the relevant states for the decay of the electric field are those in the lowest Landau level.

#### IV. PAIR PRODUCTION IN EXPANDING GLASMA

Up to now we have discussed the pair production in the system where the electric and magnetic fields are infinitely extending in space. This does not correspond to the case of the real glasma, which are expanding after the heavy ion collisions. Thus we now proceed to show how  $E$  and  $N$  behave in a system where the fields and particles expand. Namely, we calculate their behaviors in  $\tau$  and  $\eta$  coordinates in which the background field  $E_\eta$  is uniform in  $\eta$ . The behaviors of the fermions in the transverse directions specified by the coordinates  $x$  and  $y$  are identical to those in Cartesian coordinates. They are frozen in the states of the lowest Landau level.

We first rewrite the Dirac equation,  $(\gamma^0(i\partial_t - eA_0) + \gamma^3(i\partial_z - eA_3))\psi = 0$  by using the formulas,

$$\partial_0 = \partial_0\tau \partial_\tau + \partial_0\eta \partial_\eta = \cosh\eta \partial_\tau - \frac{\sinh\eta}{\tau} \partial_\eta, \quad \partial_z = \partial_z\tau \partial_\tau + \partial_z\eta \partial_\eta = -\sinh\eta \partial_\tau + \frac{\cosh\eta}{\tau} \partial_\eta \quad (11)$$

and

$$A_0 = \partial_0\tau A_\tau + \partial_0\eta A_\eta = \cosh\eta A_\tau - \frac{\sinh\eta}{\tau} A_\eta, \quad A_z = \partial_z\tau A_\tau + \partial_z\eta A_\eta = -\sinh\eta A_\tau + \frac{\cosh\eta}{\tau} A_\eta. \quad (12)$$

Then, we obtain

$$\left(\gamma^0(i\partial_\tau - eA_\tau) + \frac{1}{\tau}\gamma^3(i\partial_\eta - eA_\eta)\right)\phi = 0 \quad (13)$$

where we have set  $\psi \equiv U\phi/\sqrt{\tau}$  with  $U \equiv \cosh\frac{\eta}{2} + \gamma^0\gamma^3 \sinh\frac{\eta}{2}$ .

We solve the equation(13) with  $A_\tau = 0$  by assuming that  $A_\eta$  depends only on  $\tau$ . The assumption corresponds to the fact that the glasma is homogeneous in  $\eta$ . The solutions are given by

$$\phi_\pm = \int \frac{dk_\eta}{2\pi} \frac{A_k^\pm}{\sqrt{2}} \exp(ik_\eta\eta) v_\pm f_k^\pm \quad \text{with} \quad f_k^\pm = \exp(\mp ik_\eta \log \tau \mp i\Omega(\tau)) \quad \text{and} \quad v_\pm = \begin{pmatrix} 1 \\ 0 \\ \pm 1 \\ 0 \end{pmatrix}, \quad (14)$$

where  $A_k^\pm$  are constants and  $\Omega$  satisfies  $\partial_\tau\Omega = eA_\eta/\tau$ . We have taken into account of the fact that  $\phi$  or  $\psi$  represents only the states in the lowest Landau level.

Using these solutions, we define the field operator corresponding to the field operator in eq(2),

$$\hat{\phi} = \int \frac{dk_\eta}{2\sqrt{2}\pi} \left( \exp(ik_\eta\eta) A_k \left( \theta(k_\eta) v_+ f_k^+ + \theta(-k_\eta) v_- f_k^- \right) + \exp(-ik_\eta\eta) B_k^\dagger \left( \theta(k_\eta) v_+ f_{-k}^+ + \theta(-k_\eta) v_- f_{-k}^- \right) \right) \quad (15)$$

where the first terms ( $\propto \exp(ik_\eta - i|k_\eta| \log \tau)$ ) represent the solutions of the positive frequency and the second ones ( $\propto \exp(-ik_\eta + i|k_\eta| \log \tau)$ ) do the solutions of the negative frequency.  $A_k$  and  $B_k$  satisfy the commutation relations,

$$\{A_k, A_{k'}^\dagger\} = \{B_k, B_{k'}^\dagger\} = 2\pi\delta(k_\eta - k'_\eta), \quad \text{the other commutators vanish.} \quad (16)$$

Using the commutation relations, we find

$$\{\hat{\phi}_a(\eta, \tau), \hat{\phi}_b^\dagger(\eta', \tau)\} = \delta_{a,b}\delta(\eta - \eta'), \quad (17)$$

with  $a, b = 1, \text{ or } 3$ .

Now, we express the chiral anomaly in  $\tau$  and  $\eta$  coordinates. It is easy to see that

$$\partial_0 J^{5,0} + \partial_3 J^{5,3} = \partial_\tau J^{5,\tau} + \frac{1}{\tau} J^{5,\tau} + \partial_\eta J^{5,\eta} = 2s_\eta E_\eta \quad (18)$$

with  $J^{5,a} = \langle \bar{\psi} \gamma^a \gamma^5 \psi \rangle$  ( $a = 0, 3$ ) and  $s_\eta \equiv e^2 B_\eta / 4\pi^2$ , where the last term  $\partial_\eta J^{5,\eta}$  is assumed to vanish corresponding to the fact that the glasma is homogeneous in  $\eta$ . The current  $J^{5,\tau}$  can be represented in terms of  $J^{5,0}$  and  $J^{5,3}$  in the following,

$$J^{5,\tau} = \partial_0 \tau J^{5,0} + \partial_3 \tau J^{5,3} = \cosh \eta J^{5,0} - \sinh \eta J^{5,3} \quad (19)$$

Hereafter, we consider only the region of the central rapidity  $\eta \simeq 0$ . Then, we have

$$J^{5,\tau} \simeq J^{5,0} = \langle \frac{1}{\tau} \hat{\phi}^\dagger \gamma^5 \hat{\phi} \rangle. \quad (20)$$

In addition to the anomaly equation, we exploit a Maxwell equation,  $\partial_0 F^{0,3} = J^3$ , in order to take into account of the back reaction of the charged particles on  $E_\eta$ . The equation is represented in  $\tau$  and  $\eta$  coordinates as

$$\partial_\tau (\tau F^{\tau,\eta}) = \partial_\tau \left( \frac{F_{\tau,\eta}}{-\tau} \right) = \partial_\tau E_\eta = -\tau J^\eta \simeq -J^3 = -\langle \frac{e \hat{\phi}^\dagger \gamma^0 \gamma^3 \hat{\phi}}{\tau} \rangle, \quad (21)$$

where we have taken only the region of the central rapidity.

Therefore, we have the following equations to find the temporal behaviors of the electric field  $E_\eta$  and the number density of positrons or electrons,

$$\partial_\tau J^{5,\tau} + \frac{1}{\tau} J^{5,\tau} = 2s_\eta E_\eta \quad \text{and} \quad \partial_\tau E_\eta = -J^3 = -\langle \frac{e \hat{\phi}^\dagger \gamma^0 \gamma^3 \hat{\phi}}{\tau} \rangle, \quad (22)$$

with  $J^{5,\tau} = \langle \frac{1}{\tau} \hat{\phi}^\dagger \gamma^5 \hat{\phi} \rangle$ , where we have assumed that the electric field is parallel to the magnetic field  $\vec{E} \cdot \vec{B} = E_\eta B_\eta > 0$ . When the electric field is anti-parallel to the magnetic field, the anomaly equation becomes  $\partial_\tau J^{5,\tau} + \frac{1}{\tau} J^{5,\tau} = -2s_\eta E_\eta$ .

In the above equations the expectation values of the currents  $J^{5,\tau}$  and  $J^3$  are taken by assuming appropriate states of positrons and electrons such that,

$$J^{5,\tau} = \langle \frac{1}{\tau} \hat{\phi}^\dagger \gamma^5 \hat{\phi} \rangle = \int \frac{dk_\eta}{2\pi\tau} \left( (n_k - \tilde{n}_k) \theta(k_\eta) - (n_k - \tilde{n}_k) \theta(-k_\eta) \right) \quad (23)$$

$$J^3 = \langle \frac{e \hat{\phi}^\dagger \gamma^0 \gamma^3 \hat{\phi}}{\tau} \rangle = \int \frac{dk_\eta}{2\pi\tau} e \left( (n_k - \tilde{n}_k) \theta(k_\eta) - (n_k - \tilde{n}_k) \theta(-k_\eta) \right), \quad (24)$$

where the state  $|\rangle$  is supposed to satisfy

$$\langle A_k^\dagger A_{k'} \rangle = 2\pi\delta(k_\eta - k'_\eta) n_k(t), \quad \langle B_k^\dagger B_{k'} \rangle = 2\pi\delta(k_\eta - k'_\eta) \tilde{n}_k(t) \quad \text{and} \quad \langle A_k^\dagger B_{k'}^\dagger \rangle = \langle B_k A_{k'} \rangle = 0. \quad (25)$$

The quantities  $\frac{n_k}{\tau}$ , and  $\frac{\tilde{n}_k}{\tau}$  denote the momentum distributions of positrons and electrons, respectively. This can be easily understood by noting that the electric charge density is given by  $J^0 \equiv \frac{\langle e\vec{\phi}\gamma^0\hat{\phi} \rangle}{\tau} = \int \frac{dk_\eta}{2\pi\tau} e(n_k - \tilde{n}_k)$ . Obviously,  $J^0 = 0$  since electric charge is conserved and the initial state has no electrons and positrons.

We should make a comment that when the electric field is parallel to the magnetic field, i.e.  $\vec{E} \cdot \vec{B} = E_\eta B_\eta > 0$ , positrons ( electrons ) in the pair production have momentum  $k_\eta > 0$  (  $k_\eta < 0$  ) owing to the acceleration by the electric field. On the other hand, when  $\vec{E} \cdot \vec{B} = -E_\eta B_\eta < 0$ , positrons have  $k_\eta < 0$  and electrons do  $k_\eta > 0$ . That is,

$$(n_k - \tilde{n}_k)\theta(k_\eta) = n_k\theta(k_\eta) \neq 0 \quad \text{and} \quad (n_k - \tilde{n}_k)\theta(-k_\eta) = -\tilde{n}_k\theta(-k_\eta) \neq 0 \quad \text{for} \quad \vec{E} \cdot \vec{B} > 0, \quad (26)$$

$$, \quad (n_k - \tilde{n}_k)\theta(k_\eta) = -\tilde{n}_k\theta(k_\eta) \neq 0 \quad \text{and} \quad (n_k - \tilde{n}_k)\theta(-k_\eta) = n_k\theta(-k_\eta) \neq 0 \quad \text{for} \quad \vec{E} \cdot \vec{B} < 0. \quad (27)$$

Therefore, we find from these equations that the chirality  $J^{5,\tau}$  is given by

$$J^{5,\tau} = N + \tilde{N} \quad \text{for} \quad \vec{E} \cdot \vec{B} > 0 \quad J^{5,\tau} = -(N + \tilde{N}) \quad \text{for} \quad \vec{E} \cdot \vec{B} < 0, \quad (28)$$

where  $N = \frac{\int dk_\eta n_k}{2\pi\tau}$  and  $\tilde{N} = \frac{\int dk_\eta \tilde{n}_k}{2\pi\tau}$  represent the number densities of positrons and electrons, respectively.

Therefore, we obtain the following equations governing the pair productions,

$$\partial_\tau N + \frac{N}{\tau} = s_\eta E_\eta \quad \text{and} \quad \partial_\tau E_\eta = -2eN \quad (29)$$

where we have taken into account of the fact that the number of positrons is equal to that of electrons in the pair production,  $N = \tilde{N}$ .

We can explicitly solve the equations(29) to obtain the temporal behaviors of the electric field  $E_\eta$  and the number density of the charged particles. It is easy to see that  $E_\eta$  satisfies

$$\partial_\tau^2 E_\eta + \frac{1}{\tau} \partial_\tau E_\eta + 2es_\eta E_\eta = 0, \quad (30)$$

and  $N$  is given by  $N = -\partial_\tau E_\eta / 2e$ . We obtain the solutions with the initial conditions,  $E_\eta(\tau = 0) = E_0$  and  $N(\tau = 0) = 0$ ,

$$E = E_0 J_0(\sqrt{2es_\eta}\tau) \quad \text{and} \quad N = \frac{E_0 \sqrt{2es_\eta}}{2e} |J_1(\sqrt{2es_\eta}\tau)| \quad (31)$$

where  $J_{0,1}$  denotes Bessel functions. We have shown the behaviors of these quantities in Fig.2. We can see that they decrease oscillating with time  $\tau$ , while they simply oscillate without the decrease in the ordinary Cartesian coordinates ( Fig.1 ). The decrease comes from the expansion of the glasma.

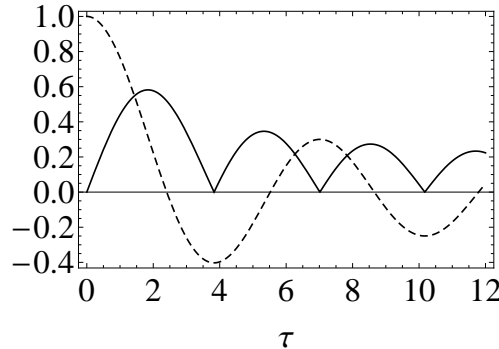


FIG. 2: electric field  $E(\tau)$  (dashed line) and number density of electrons  $N(\tau)$  (solid line) with arbitrary scale

The implication of the figure is in the following. When the electric field  $E_\eta > 0$  is switched on, the pair production arises and the number density  $N$  increases, while the field becomes weak owing to the energy lose. The energy lose is

caused from the acceleration of the charged particles. There are two effects which make the number density increase or decrease. The number density increases by the pair production ( $s_\eta E_\eta$ ), while the expansion ( $-N/\tau$ ) makes the number density decrease:  $\partial_\tau N = s_\eta E_\eta - N/\tau$ . At the beginning of the pair production,  $N$  increases because the effect of the pair production is stronger than that of the expansion. But, at the time  $\tau = \tau_0$ ,  $N$  stops increasing because both effects balance with each other. After that, the effect of the expansion becomes stronger than that of the pair production. Thus,  $N$  begins to decrease. On the other hand, the electric field loses its energy with the acceleration of the particles and vanishes at the time  $\tau = \tau_1 > \tau_0$ ;  $E_\eta(\tau = \tau_1) = 0$ . Then, the field changes its direction, i.e.  $E_\eta < 0$ . The particles begin to be accelerated in the direction opposite to their velocity  $k_\eta > 0$ . Thus, the pair annihilation begins and makes the number density decrease furthermore. Because the electric field accelerates the particles into the direction opposite to the particle velocity, the field gains the energy from the particles. Thus, the field becomes strong. Eventually the number  $N$  of the particles vanishes ( $N(\tau = \tau_2) = 0$ ) at the time  $\tau = \tau_2 > \tau_1$  when the annihilation stops and the increase of the field strength also stops. We can see that at the time  $\tau = \tau_2$  a new pair production begins to take place under the effect of the electric field  $E_\eta(\tau = \tau_2) < 0$ . This is the physical explanation of the behaviors depicted in the figure 2.

We should also mention that the life time  $\tau_1 \simeq 2.4/\sqrt{2es_\eta}$  (the first zero point of the field  $E \propto J_0(\sqrt{2es_\eta}\tau)$ ) of the electric field is larger than the corresponding one  $t_1 \equiv \pi/2\sqrt{2es}$  of  $E \propto \cos(\sqrt{2es}t)$  in the non-expanding glasma with  $s = s_\eta$  or  $B = B_\eta$ . Since the expansion makes the number density of the charged particles after their production lower than that in the non-expanding case, the rate of the energy loss of the field is slower than that in the non-expanding case. Hence, the life time  $\tau_1$  is larger than  $t_1$ .

## V. SUMMARY AND DISCUSSION

To summarize, we have shown that due to the pair production of the charged particles, the electric field homogeneous in  $\eta$  decreases with the oscillation in the  $\tau$  and  $\eta$  coordinates. This should be contrasted with the case in the Cartesian coordinate where the field homogeneous in  $z$  simply oscillates without the decrease. The number density of the electrons shows the similar behavior as that of the electric field. Since  $\tau$  describes a proper time associated with the expanding fluid, it is natural that the energy of the electric field and the number density become lower with time  $\tau$  for  $\eta$  fixed. This lower number density of the charged particles leads to the longer life time of the electric field. We expect that these behaviors shown in QED also arise in the real glasma produced in high energy heavy ion collisions.

We have only discuss the quark pair production under the color electric field produced in high energy heavy ion collisions. In the collision the most important products would be gluons, which can be also produced in the Schwinger mechanism[19]. In order to discuss the gluon production, we need to take into account of the effect of the color magnetic field. As is well known, the gluons are unstable under the magnetic field. It means that the gluons are coherently produced[20]. Just as Higgs field  $\phi_H$  located at the top of the potential  $\phi_H = 0$ , the gluon fields exponentially grow to approach the bottom of the potential. The process can be regarded as the coherent production of the gluons. It is important to find the ratio of the number density of the gluon to that of the quarks produced in the Schwinger mechanism. In near future, we discuss the gluon production as well as the quark production by taking account of the back reaction of the coherent gluons and the quarks.

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